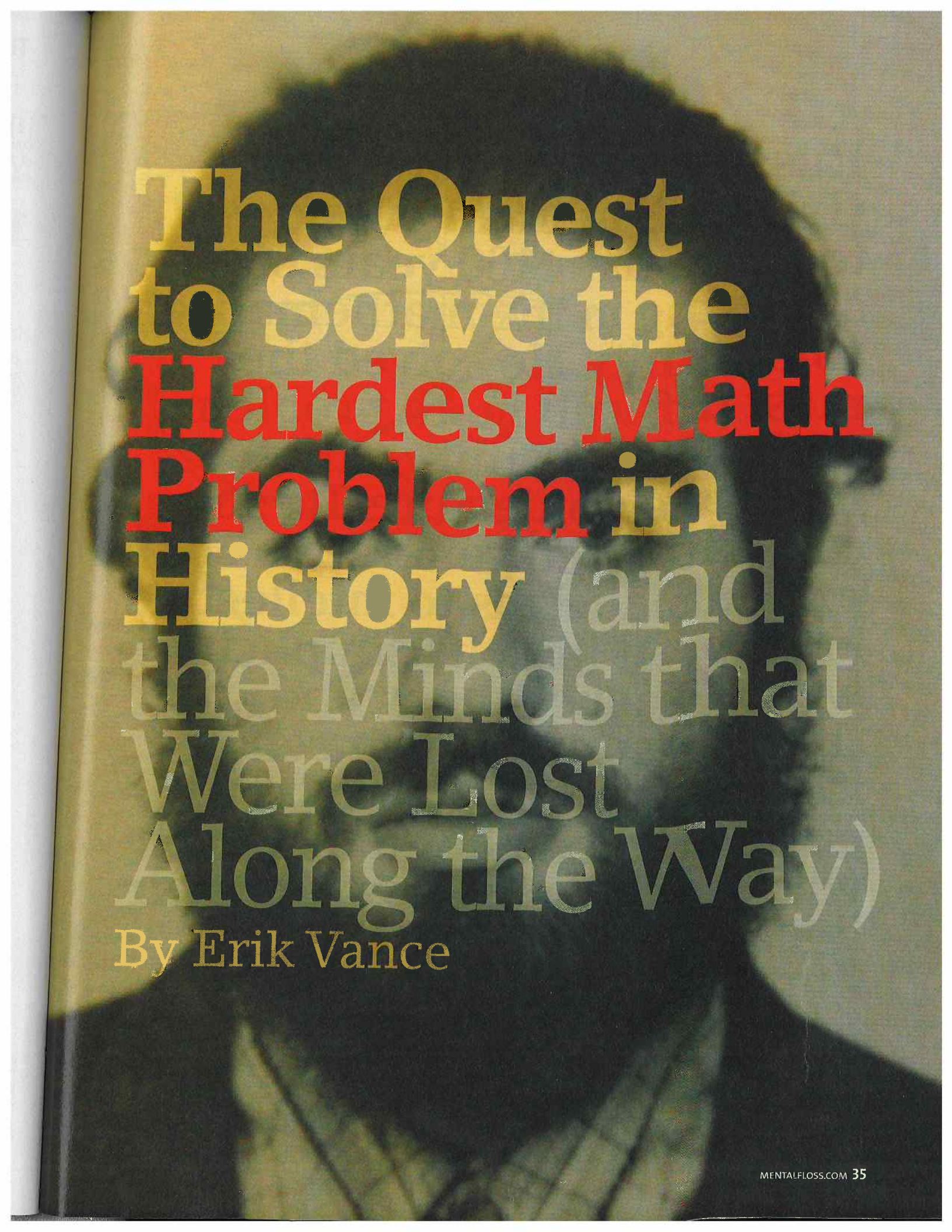
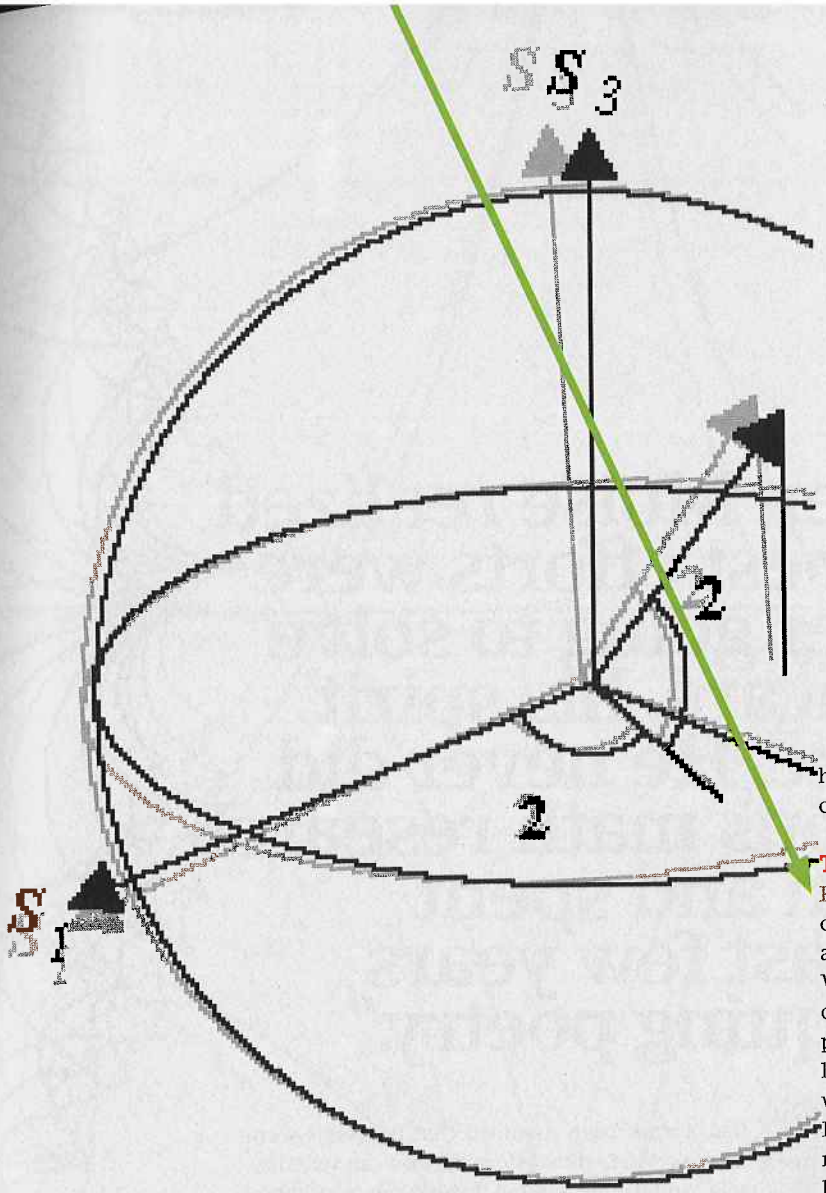


The Quest to Solve the Hardest Math Problem in History (and the Minds that Were Lost Along the Way)

By Erik Vance

In 2002, a reclusive Russian genius named Grigori Perelman put an end to more than 100 years of suffering in the mathematical community. He solved the most difficult math problem of the 20th century—the Poincaré Conjecture. Its siren call had lured generations of mathematicians to intellectual graves. At first, its simplicity would seduce them, and they'd become convinced the answer was near. But as years passed, they'd be left with nothing to show for their lives' toil but dead ends. By the time Grigori Perelman proved the Conjecture, the solution was worth $1 million.



THE MAN BEHIND THE MADNESS

In 1885, all of Europe was talking about Henri Poincaré, a 30-year-old genius who'd mathematically proven why the solar system holds together. When a hole appeared in his calculations, he plugged it up by essentially inventing chaos theory. Kings were tripping over themselves to make him a knight, and Sweden gave him a small fortune in prize money. To this day, Poincaré holds the record for the most physics Nobel Prize nominations, though he never actually won one.

But his most legendary achievement was something no one noticed until much, much later. At the turn of the century, Poincaré invented an entirely new field called algebraic topology, and today, it's one of the most complicated and vibrant branches of mathematics. Think of it as a twisted version of geometry, in which shapes stretch, bend, and fold inside out. Poincaré's goal was to classify objects by identifying their basic form, much the same way botanists classify new species of plants. In the process of creating topology, Poincaré tossed out a conjecture that seemed to be true. It was a side note to a larger problem, and he figured he'd work out the details later. Little did he know, his side note would become one of the greatest challenges in the mathematical world.

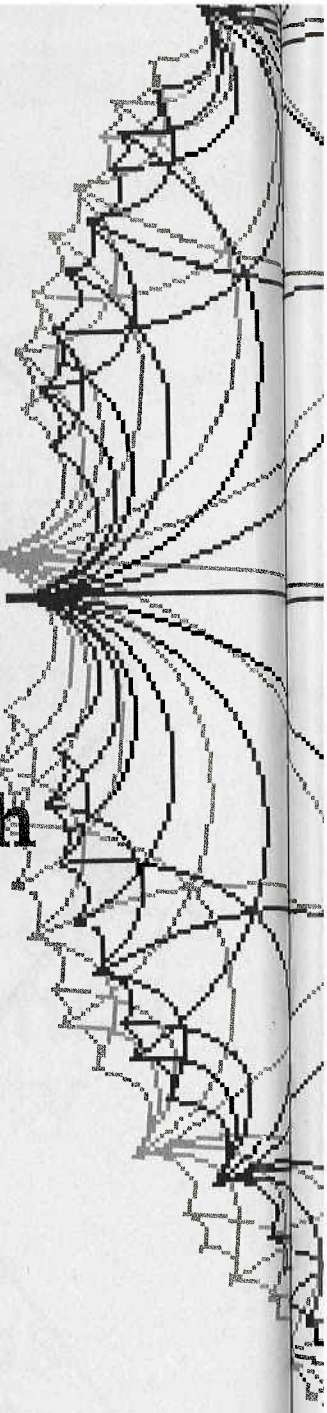
THE VICTIMS

Poincaré's conjecture seemed simple enough. It claimed that any object without a loop is essentially a sphere. Think of a knife made out of Play-Doh. Without punching a hole in it or closing a loop, can you squish it into a ball? Yes, of course. Now picture a pair of Play-Doh scissors. No matter how hard you try, you can't crush it into a ball without closing up the finger holes. It's impossible. Poincaré believed that objects like the knife were related to spheres, while objects with holes and loops in them were not.

Poincaré thought the conjecture would be easy to prove, and he even published a solution. But then, he saw a flaw in his work and retracted it. After his death in 1912, the question lay dormant for decades, until an Oxford professor named J.H.C. Whitehead rediscovered it in the late 1930s. J.H.C. (known to his students as "Jesus, he's confusing") also published a solution. But he, too, found a mistake and retracted it. However, his work sparked interest in the problem. By the 1950s, the Poincaré Conjecture was one of the best-known challenges in the math community.

That's when two Princeton students, Edwin Moise and Christos Papakyriakopoulos (commonly known as Papa), decided to try their hands at it. Moise in particular looked like the guy to do it. Young and brash, he liked to announce his next big problem like a batter calling his shot. Twice that included one of the toughest problems in topology, and twice he returned with the solution. Then, he set his sights on Poincaré.

Papa was vastly different. A self-taught political refugee from Greece, he was famous for his odd, obsessive nature. Legend has it that when he came to Princeton, he checked into a motel and



"When Moise realized his best efforts were never going to solve Poincaré, his spirit broke. He never did serious math research again and spent his last few years critiquing poetry."

never checked out. He never even unpacked his bags. He simply fell into a routine that he followed every day, down to the minute, which always included a midday nap on top of his desk.

Throughout the 1950s, the two geniuses dueled with each other over Poincaré. Papa would announce a proof, and Moise would shoot it down. Then Moise would announce a proof, and Papa would shoot it down. This went on for years, while neither man worked on almost anything else.

Eventually, Moise cracked. One day, he simply turned away from math altogether. Michael Freedman, a topologist who works for Microsoft, describes this phenomenon as getting "wrecked" by a problem. He says many mathematicians thrive on the knowledge that they are smart enough to solve almost anything, given enough time. When Moise realized his best efforts were never going to solve Poincaré, his spirit broke. He never did serious math research again and spent his last few years critiquing poetry. Papa kept working on the problem for 25 years, swearing he wouldn't marry until he'd solved the Conjecture. In 1976, he died of stomach cancer, still a bachelor.

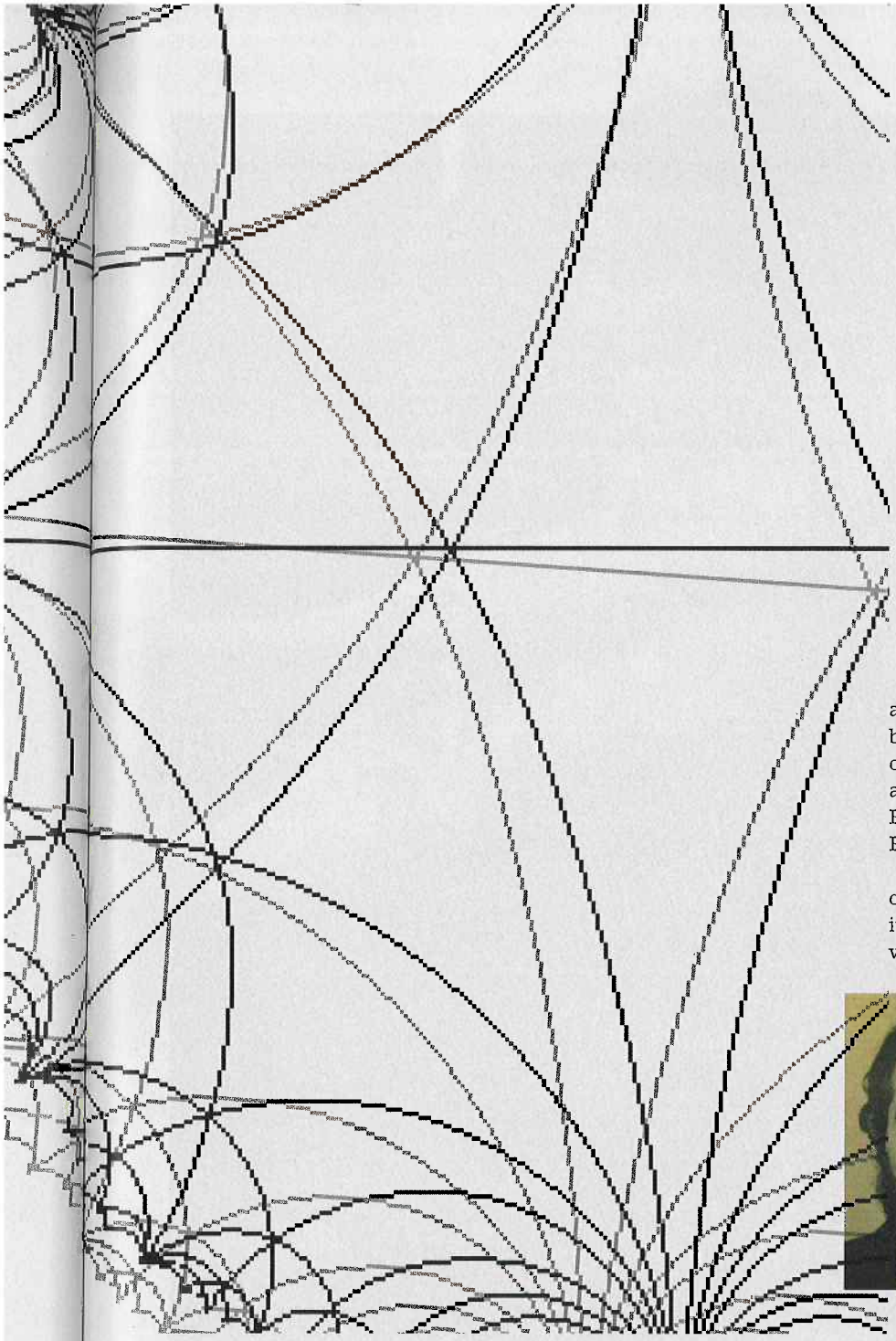
HOPE IN A HIGHER DIMENSION

After dozens of mathematicians had devoted their careers to the Poincaré Conjecture, a breakthrough came in 1960, when a young hotshot named Steven Smale made the first tangible headway into the problem. Smale decided not to worry about objects in the three-dimensional or even the four-dimensional universe. Instead, he proved that the Conjecture was true in the fifth dimension and higher. Until then, it had always been assumed that problems were easier to solve in dimensions that we can visualize. Smale broke new ground by solving a problem in higher dimensions *before* the lower ones, and today, it's common practice. Mathematicians say the extra dimensions give them room to manipulate imaginary objects.

Smale's discovery inspired the math world, and a new generation of Don Quixotes started sharpening their lances. Another ray of hope came in 1982, when mathematician Michael Freedman managed to scoop up Poincaré in the fourth dimension. Both he and Smale received the Fields Medal, math's equivalent to the Nobel Prize, just for their partial proofs. And yet, the question of the third dimension—the only one that had actually interested Henri Poincaré—still remained. Technically speaking, without the third dimension, mathematicians were no closer to the answer than they'd been in 1904.

TO THE VICTOR GO NONE OF THE SPOILS

Through all of this, some mathematicians devoted themselves to *disproving* Poincaré. In fact, during the 1950s, a man named R.H. Bing would spend two weeks trying to prove the Conjecture was true and then two weeks trying to prove it was



false. Neither effort panned out. Regardless, most mathematicians believed the solution to Poincaré was out there—somewhere.

Finally, in 2002, Grigori Perelman, a recluse living with his mother in St. Petersburg, posted a short paper on a math Web site. The reticent Perelman never once mentioned Poincaré in his essay, but the few people who read it understood its implications. The paper addressed one of the biggest obstacles that had blocked mathematicians from proving the Conjecture. Whenever they'd tried reducing certain shapes to their most basic forms, little irregularities kept popping up like painful burrs. To smooth out the rough spots, Perelman applied a type of mathematical sandpaper called a Ricci flow. The math community started to buzz that he'd resolved the underlying issues of the problem. Eventually, one mathematician asked Perelman directly if his paper answered the Poincaré Conjecture. Never one to be long-winded, Perelman wrote back, "That is correct."

Because Perelman hadn't taken the normal steps of running his ideas past colleagues and publishing in a referenced journal, it took some time to verify this claim. It didn't help that his paper only included the barest essentials to prove the Conjecture. It took six experts two years to fill in the gaps that Perelman had seen as self-evident.

GRIGORI PERELMAN

By 2006, his proof had held up to every attack, and Perelman was offered a Fields Medal. But by that time, Perelman was so disillusioned with the field of mathematics that without a flourish, he turned it down, becoming the first person in history to refuse the award. Also without explanation, Perelman never claimed the $1 million prize from the Clay Mathematics Institute in Cambridge, Mass.

Today, Poincaré's side note may only be applicable to the most obscure physics problems. But mathematicians expect that, like most theoretical breakthroughs, the effects will eventually diffuse to the rest of science. After all, when Newton first unveiled calculus, experts said only a few people on Earth could possibly understand it. Now, it's taught to teenagers and used in everything from engineering to statistics. Whether Grigori Perelman likes it or not, someday he may find himself honored in high school textbooks, and the Poincaré Conjecture may become as easy to understand as gravity.

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